Government Subsidies, ESG Performance, and Common Ownership*

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Abstract

We construct a continuous-time multi-player game model where N firms compete in the market, while the government incentivizes entrepreneurs to actively fulfill their environmental, social, and governance (ESG) responsibilities by offering competitive financial subsidies. In this model, risk-neutral agents hired by principals can influence the projects' operation revenues and their negative externalities by determining ESG performance. We derived the contractual incentive relationship within the company and the optimal competitive ESG incentive policy. We also provide examples of security implementation and conduct a comparative static analysis of the optimal contract. Finally, we consider the potential effects of common ownership by institutional investors and conduct a comparative analysis. Our findings indicate that, when the total amount of the government's ESG incentives is fixed, the synergistic governance effect of common ownership improves total ESG performance. However, when the government implements the theoretical optimal incentive policy, both the collusive fraud effect and synergistic governance effect lead to a decline in total ESG performance. Hence, fixing the total amount of subsidies might be a better solution for governments to incentivize companies' ESG activities.

Keywords: ESG, Contract design, Externalities, Common ownership.

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1. Introduction

In line with the increasing concerns about climate change, the concept of environment, social, and governance (ESG) and sustainability has gained substantial attention from governments, companies, and investors (Avramov et al., 2022; Pedersen et al., 2021; Bénabou and Tirole, 2010). Governments worldwide are making commitments to achieve carbon neutrality so as to prevent climate-related catastrophes (Masood, 2021)[1]. Specifically, several governments have initiated national-level legislative efforts to mandate ESG compliance, such as requiring listed companies to disclose ESG/CSR reports (Subramaniam et al., 2017). Global investors also shift their strategies to sustainable investing and consider ESG factors in their portfolio selection and management processes (Engle et al., 2020). Such a strategic shift results in an exponential growth of ESG investment in the global financial market,[2] thus facilitating firms to enhance their ESG performance and improve their ESG disclosure practices (Christensen et al., 2021). For some investors, ESG investment primarily expresses their nonpecuniary preferences. In contrast, for other investors, ESG investment means considering the ESG aspects of the firm to mitigate risks and enhance returns (Starks, 2023). Within this context, firms face continued pressure to conduct their operations sustainably such as improving their ESG performance (Habermann and Fischer, 2023).

Nevertheless, investing in ESG is a long-run process, and the associated costs also require a long time to yield financial returns for the company. In a traditional agency-theoretic framework, corporate owners only care about a company's financial performance, and omit broader societal measures such as ESG performance. As the agent's compensation is irrelevant to the firm's ESG performance, rational rational agents might lack the motivation to engage in ESG investments. To avoid such circumstances, incentivizing agents to actively embrace ESG investments is both essential and urgent. One incentive strategy is incorporating ESG metrics into compensation contracts. As shown in Cohen et al. (2023), the

[1]A report released by Energy & Climate Intelligence Unit in 2021 reveals that 124 countries out of 202 surveyed have made commitments to achieve net-zero emissions. See, https://ca1-eci.edcdn.com/reports/ECIU-Oxford_Taking_Stock.pdf?v=1616461369

[2]Since the launch of the United Nations Principles for Responsible Investment (PRI) in 2006, the number of signatories has grown from 734 in 2010 to 3826 in 2021, with total assets under management of US$ 21 trillion in 2010, and US$ 121.3 trillion in 2021. See, https://www.unpri.org/about-us/about-the-pri

percentage of companies using ESG metrics as executive key performance indicators (KPIs) for executives has grown from 3% in 2010 to over 30% in 2021, and the adoption of ESG Pay is accompanied by improvements in key ESG outcomes. Additionally, as mentioned in the McKinsey Quarterly's November 14, 2019 report, "Five Ways ESG Creates Value" lists earning subsidies and government support as part of the value created by pursuing ESG strategies [3]. Another source of incentives could be the governments intervention (subsidies). However, limited literature has provided methodology support for the government's ESG incentives under optimal contract theory. Therefore, it is necessary to theoretically solve the problems of how governments formulate incentive decisions, how these incentives are transmitted to companies, and how the incentives influence the agents' choices regarding ESG investments.

In this paper, we follow Williams (2015) and propose a continuous-time multi-player game model that includes N firms in a competitive market and a government as an ESG subsidy provider. We use a piecewise second-order ordinary differential equation to describe the contractual incentive relationship within an enterprise. In our model, we assume that the operations of firm projects create negative externalities, and a firm's ESG performance can mitigate the reduction in economic benefits caused by externalities. At the same time, the high cost of ESG activities can reduce the company's cash flow. We examine the incentive strategies offered by the government and employ the stochastic maximum principle approach to address the contract problem between investors and professional managers. We then derive the optimal competitive ESG inventive policy, as well as the analytical formula for the optimal ESG inventive factor and ESG performance.

Turning to the role of investors, over the past three decades, the share of public U.S. firm stock owned by large institutional investors has increased substantially and become more concentrated (Park et al., 2019). Particularly, these largest investors each own 5% to 7% of almost all companies in the S&P 500 (Schmalz, 2018). This has led to the phenomenon of common ownership, in which institutional investors hold large stakes (at least 5% ownership) in at least two companies in the same industry. There are two opposing perspectives on

[3] See, https://www.mckinsey.com/capabilities/strategy-and-corporate-finance/our-insights/five-ways-that-esg-creates-value

the impact of common ownership on firms' ESG engagement. The collusive fraud effect argument suggests that when investors have (partial) equity ownership in firms, they may exert pressure on managers to internalize the externalities of competitive behavior in order to maximize portfolio value (Azar, 2012), thereby, these firms have less incentive to compete and greater incentive to consider the benefits of their actions for commonly-owned peer firms (Azar et al., 2018, 2022), i.e., have less incentive to compete for government subsidies by investing in ESG activities. The synergistic governance effect viewpoint suggests that common institutional ownership can promote market coordination among competitors, and internalize governance externalities among peers by alleviating free-riding concerns, leading to greater gains from ESG engagement (Cheng et al., 2022; He and Huang, 2017). Moreover, pro-social common ownership may be more likely to drive firms to engage in ESG activities (Chen et al., 2020; Dyck et al., 2019). Hence, another important question arises: how does the common ownership affect the relationship between government subsidies and the firms' ESG performance?

To answer this question, we compare the total ESG performance of companies with common ownership from two perspectives. Sections 4.1 and 4.2 show the results of the impact of common ownership on the degree of government subsidies' incentive on firms' ESG performance under the collusive fraud effect and synergistic governance effect, respectively. Our results suggest that when the total amount of the government's ESG incentives is fixed, the synergistic governance effect of common ownership leads to an increase in the total ESG performance. However, when the government implements the theoretical optimal incentive policy, the collusive fraud, and synergistic governance effects lead to a decline in total ESG performance.

The main contributions of our paper are as follows. First, our paper contributes to the literature on contract theory. Prior studies mainly focus on maximizing the participants' own utility under the constraints of participation and incentive compatibility (DeMarzo et al., 2012; Williams, 2015; Zhu, 2022). We extend this literature by introducing ESG input decisions into the traditional contract design problem by developing a stochastic process that captures externalities.

Second, this study contributes to the literature on common ownership. Existing empirical studies have focused on the economic implications on corporate governance and shareholders, such as improving corporate governance (He et al., 2019) and reducing earnings management (Ramalingegowda et al., 2021). Cheng et al. (2022) find that common ownership is beneficial for shareholders, but it can also reduce the welfare of stakeholders. Our paper finds that when the government implements the theoretical optimal incentive policy, the existence of common ownership will lead to a reduction in total ESG performance in the competitive market. Consistent with findings of Cheng et al. (2022), we provide theoretical support for the viewpoint that common ownership may reduce the welfare of stakeholders.

Third, this paper contributes to the literature on ESG performance. Prior literature has documented that a firm's ESG profile and activities are strongly related to the firm's market characteristics (Cai et al., 2016; Di Giuli and Kostovetsky, 2014), executive characteristics(Borghesi et al., 2014), executive compensation (Ferrell et al., 2016; Flammer et al., 2019), ownership characteristics (Dyck et al., 2019; Abeysekera and Fernando, 2020), and firm risk (Albuquerque et al., 2019; Zerbib, 2019). However, current research on firms' ESG activities mainly focuses on empirical studies. Our paper, which combines ESG and contract theory, can provide theoretical support for understanding the relationship between management characteristics and ESG activities, as well as reveal the underlying mechanisms.

Fourth, our study provides implications for policymakers, investors, and managers. For policymakers, fixing the total amount of subsidies in advance and letting companies compete for the subsidies may be a better solution when setting ESG incentives. For institutional investors with common ownership, they can promote cooperation between firms on environmental and social issues, thereby alleviatubg information asymmetry, improving their own economic benefits, promoting the development of companies in the indusry and increasing the social welfare of stakeholders. In addition, we also provides theoretical guidance for firm managers when making decisions on ESG performances in a competitive market.

The remainder of the paper is organized as follows. Section 2 sets up the model. Section 3 considers optimal contract and ESG incentives under oligopoly competition. Section 4 solves the optimal contract and ESG incentives under common ownership. Section 5 provides an implementation of the optimal contract. Section 6 provides numerical analysis to discuss how the model results are influenced by model parameters. Section 7 concludes. Proofs are relegated to the Appendix.

2. Model Setup

We develop a continuous-time multi-player game model that includes N firms and a government decision-maker in a competitive market. Job market consists of homogeneous managers who possess the same abilities and costs. Investors[4] hire managers to oversee projects within their respective companies. The model involves a standard moral hazard problem in project management, where the agent may reduce effort to save on costs, thereby decreasing the expected profitability of the project.

In contrast to traditional principal-agent models, we allow agents to make decisions about the firm's ESG performance, which reduces externalities resulting from the company's operating activities in a linear form. Additionally, the government incentivizes competitive ESG performance and collects corporate income tax at a rate of τ. Our dynamic principal-agent model operates in continuous time, where the (cumulative) cash flow of company-n, $n = 1, ..., N$, denoted by X_n, is driven by a common standard Brownian motion Z_n over the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\leq t<\infty}, \mathbb{P})$ and is endowed with information flows:

$$\mathrm{d}X_{n,t} = \left[a_{n,t} + \left(P - Q\sum_{m=1}^{N}\eta_{m,t}\right)\eta_{n,t} - \theta_n\eta_{n,t}\right]\mathrm{d}t + \sigma_n\mathrm{d}Z_{n,t},\ \ X_{n,0} = x,\ \ n = 1, ..., N, \tag{1}$$

where process a_n represent the unobserved effort of the agent, with a limited range of values $\mathcal{E} = [0, \bar{a}]$, process η_n denote the value of the company's ESG performance, and θ_n represent the fixed marginal cost of the company's ESG performance. The volatility of the project's cash flow is represented by σ_n, and T_n is the project termination time. The parameters $\theta_n > 0$, $\bar{a} > 0$, and $\sigma_n > 0$ are all exogenously given constants. We assume that the manager can choose to perform the ESG responsibility to varying degrees $\eta_{n,t}$ at any time $t \in [0, T_n]$. We say $X_{n,t} \in L^2$ if $\mathbb{E}\int_0^T X_{n,t}^2\mathrm{d}t < \infty$. In a competitive market, the government provides incentives based on the ESG performance of each firm at a unit price of $\left(P - Q\sum_{m=1}^{N}\eta_{m,t}\right)$ at any time $t \in [0, T_n]$. In this context, P denotes the government's control variable, repre-

[4]In this paper, the terms "investor", "principal", and "she" are used interchangeably. Similarly, the terms professional "manager", "agent", "executive", and "he" are used interchangeably.

senting the initial incentive level in the absence of ESG performance in the market, while Q is an externally given fixed parameter that indicates how incentives are influenced by market competition.

In contrast to general dynamic principal-agent models, we assume that the project's operations create negative externalities. Economic benefits of externalities of the nth company Y_n, $n = 1, ..., N$, is accumulated at a fixed rate of $-b_n$ and is somewhat stochastic. A company's ESG performance can mitigate the reduction in economic benefits caused by externalities, and the extent of mitigation is determined by the company's ESG performance process η_n. Specifically, the externality process Y_n is given by

$$\mathrm{d}Y_{n,t} = (-b_n + \eta_{n,t})\,\mathrm{d}t + \phi_n \mathrm{d}B_{n,t},\ Y_{n,0} = y,\ n = 1, ..., N, \tag{2}$$

where B_n is the standard Brownian motion defined as independent of Z_n on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0 \le t < \infty}, \mathbb{P})$, and the constant ϕ_n represents the volatility of the project externalities.

Game and decision. In our model, the contract and policy depend on the available information to the participants. We assume that both parties can observe the project cash flows X_n, externalities Y_n, and ESG performance η_n. Denote $\mathcal{G}_{n,t} \equiv \sigma\{X_{n,s} : 0 \le s \le t\},\ n = 1, ..., N$, and $\mathbb{G}_n \equiv \{\mathcal{G}_{n,t}\}_{t \ge 0},\ n = 1, ..., N$, representing the augmented filtration generated by the cash flow X_n or the information flow learned by the participants who observe X_n. Similarly, denote $\mathcal{U}_{n,t} \equiv \sigma\{Z_{n,s} : 0 \le s \le t\}$, and $\mathbb{U}_n \equiv \{\mathcal{U}_{n,t}\}_{t \ge 0}$ represents the information flow generated by the Brownian motion Z_n. However, the agent's effort behavior a_n and stochastic drive Z_n are observable only for the agent. In brief, the process X_n is $\mathbb{G}_n$-adapted, and a_n and Z_n are $\mathbb{U}_n$-adapted. The information flow described by $\mathbb{U}_n$ includes $\mathbb{G}_n$. We emphasize that the information flow of the principal is $\mathbb{G}_n$, while the information flow of the agent is $\mathbb{U}_n$. Thus, manager have access to (weakly) more information than investor, and this difference in their access to information can lead to moral hazard. For a detailed discussion of the relevant information flows, see Chapter 5 of Cvitanic and Zhang (2012).

For each company n, $n = 1, ..., N$, we assume that the agent is essential for project operation and that the project is profitable enough on average to prevent the principal from terminating the contract. The agent determines the termination time T_n endogenously. Upon termination, the principal receives the liquidation value L_n, while the manager receives an exogenous dollar value $R_n > 0$ from an outside option.

The principal pays the agent at a compensation rate of c_n per unit of time to provide an incentive. The government policy-maker pays the agent at a rate of $\left(P - Q\sum_{m=1}^{N}\eta_m\right)\eta_n$ to incentivize ESG performance. Given a contract defined by (c_n), government incentive (P), and ESG performance of other companies in competitive markets $(\eta_m)_{m\neq n}$, the agent chooses a strategy (a_n, η_n, T_n) defined by the effort level process a_n, ESG performance η_n, and contract termination time T_n. Strategy (a_n, η_n, T_n) is called incentive-compatible if it maximizes the agent's value. The principal's problem is to design an optimal contract that the agent is willing to accept while maximizing the principal's value, and the agent takes an incentive-compatible strategy. The government policy-maker's problem is to design an optimal ESG incentive factor that maximizes the government's value.

Noting that the principal can invest their wealth in different firms to diversify risk, we assume that the government and investors are risk-neutral and discount utility based on the risk-free rate r. Similarly, we assume that the agent is risk-neutral, but their subjective discount rate γ is higher than the risk-free rate r[5]. Following Huang et al. (2022) and DeMarzo et al. (2012), we assume that the cost of effort has a linear form: $g(a_{n,t}) \equiv \lambda a_{n,t}, t \geq 0$. We assume that effort costs have the same unit measure as consumption utility, where the coefficient λ denotes the fixed marginal cost of effort.

Measure transformation. Due to the history dependence of the contract, we cannot use a direct approach to the problem of optimal contracting. The function containing the entire past history $\mathbb{G}_n$ would be a state variable. To express the problem canonically, we assume that all random variables and stochastic processes are defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Following Williams (2015), we consider the output and externality when the level of effort

[5] This assumption is common in contract theory, see DeMarzo and Sannikov (2006) and DeMarzo et al. (2012).

and ESG performance are held constant at a minimum value of 0, i.e.

$$\mathrm{d}X_{n,t} = \sigma_n \mathrm{d}Z_t^0. \tag{3}$$

According to Girsanov's theorem, the output distribution defined by (1) can be derived from (3) by a probability measure transformation. It is sufficient to redefine the probability measure on the measurable space $(\Omega, \mathcal{F})$ as

$$\mathbb{Q}(A) = \mathbb{E}\left(\Gamma_{n,T}\mathbb{I}_A\right), \ A \in \mathcal{F}, \tag{4}$$

where $\mathbb{I}_A$ is the characteristic function of the set A, and

$$\Gamma_{n,T} \equiv \exp\Bigg[\int_0^T \frac{a_{n,t} + \left(P - Q\sum_{m=1}^N \eta_{m,t}\right)\eta_{n,t} - \theta_n\eta_{n,t}}{\sigma_n}\mathrm{d}Z_t^0 - \frac{1}{2}\int_0^T \left(\frac{a_{n,t} + \left(P - Q\sum_{m=1}^N \eta_{m,t}\right)\eta_{n,t} - \theta_n\eta_{n,t}}{\sigma_n}\right)^2 \mathrm{d}t\Bigg].$$

This finding suggests that a change in the level of effort from a constant zero to a process a_n and a change in the ESG performance from a constant zero to a process η_n corresponds to a change in the distribution of outputs. This change can be interpreted to mean that the effort and ESG performance only alter the probability measure on the measurable space $(\Omega, \mathcal{F})$ $(\mathbb{P} \to \mathbb{Q})$ and do not change the possible values of the output process. The measure transformation viewpoint makes the sample function (orbit) of the process X_n independent of the agent's level of effort and ESG performance. Effort and ESG performance only change the probability distribution, allowing for an effective solution to the non-Markovian process problem encountered in the equilibrium computation of the game between the government, the principal, and the agent.

3. Optimal contract and ESG incentives under oligopoly competition

In general, there is a cost to enhancing effort and ESG performance. If ESG performance is not directly reflected in the company's project cash flows, decision makers may refuse to fulfill ESG responsibilities due to individual rationality. Therefore, the principal is required to provide incentives for the agent to enhance the effort level, and the government is required to provide incentives for the company to enhance ESG performance. Of course, incentives cannot be constant but should increase as the project's output increases. If project performance continues to deteriorate, the agent may choose to terminate the contract ($T_n < \infty$).

3.1. Agent's problem

We first consider the agent's problem. In response to a take-it-or-leave-it contract (c_n) offered by the principal, the agent needs to solve the following optimization problem:

$$\sup_{a_n,\eta_n,T_n} \mathbb{E}^{a_n,\eta_n} \left\{ \int_0^{T_n} e^{-\gamma t}[c_{n,t} - g(a_{n,t})]\mathrm{d}t + e^{-\gamma T_n} R_n \right\}, \tag{5}$$

where control process a_n is $\mathbb{U}_n$-adapted and T_n is a stopping time of the filtration $\mathbb{U}_n$. The superscripts a_n and η_n on the expectation operator emphasize that the rate of payoff c_n granted by the principal depends on the agent's level of effort and ESG performance. According to DeMarzo and Sannikov (2006), the agent does not face a savings choice due to their higher subjective discount rate.

The agent's problem involves control with stochastic coefficients in the presence of changing variables. Drawing upon the work of Williams (2015), we utilize the stochastic maximum principle proposed by Bismut (1973) to establish the necessary condition for optimality. Similar to the deterministic Pontryagin's principle, the stochastic maximum principle introduces the Hamiltonian, which encapsulates the optimality condition within its differential form. Nonetheless, given the inclusion of stochastic state variables, the accompanying variables comprise a collection of process vectors: one set of process vectors multiplied by the state drift, and another set of process vectors dot-multiplied by the state-dependent diffusion vector. This combination of accompanying processes effectively solves a system of backward stochastic differential equations (BSDEs). The Hamiltonian $\mathcal{H}$ of the agent's problem with the states pair $(W_{n,t}, \alpha_{n,t})$[6] can be defined as:

$$\begin{aligned}\mathcal{H} &= \Gamma H(a_n, \eta_n, c_n, P, Q, \tau, \theta_n, \alpha_n) \\ &= \Gamma\left[c_n - g(a_n) + \alpha_n(1-\tau)\left(a_n + \left(P - Q\sum_{m=1}^{N}\eta_m\right)\eta_n - \theta_n\eta_n\right)\right].\end{aligned} \tag{6}$$

Lemma 3.1. *Given a strategic profile $\{(c_n); (P); (\eta_m)_{m\neq n}; (a_n, \eta_n, T_n)\}$, the Hamiltonian is maximized by the optimal control and the differential of the Hamiltonian controls the evolution of the co-state. The co-state $W_{n,t}$ can be determined through the following BSDE:*

$$\begin{aligned}&\mathrm{d}W_{n,t} = (\gamma W_{n,t} - c_{n,t} + \lambda a_{n,t})\,\mathrm{d}t + \alpha_{n,t}\sigma_n(1-\tau)\mathrm{d}Z_t^{a_n,\eta_n}, \\ &W_{n,T_n} = R_n.\end{aligned} \tag{7}$$

The co-state $W_{n,t}$, as shown in (7), represents the sum of the agent's discounted utility from time $t \in [0, T_n]$ until the termination of the project T_n. This quantity is commonly referred to as the continuation value, or commitment utility.

Incentive-compatible condition of contract. The co-state aligns with the unique state variable proposed by Sannikov (2008) from a martingale perspective. We make the assumption that it is inefficient for the agent not to exert their maximum effort $\bar{a}$. Therefore, following the Revelation Principle (Myerson, 1979), we can consider only the admissible contract defined below, without any loss of generality.

Definition 3.2. *For $0 \leq t \leq T_n$, a contract (c_n) is considered admissible if its corresponding agent's incentive-compatible strategy (a_n, η_n, T_n) satisfies $a_{n,t} = \bar{a}$ and $\eta_{n,t}$ maximizes the company's cash flow.*

Intuitively, to make a contract admissible, the principal must provide the agent with sufficient incentives relative to the effort cost. Formally, we have

Proposition 3.3. *Given a strategy profile of $\{(c_n); (P); (\eta_m)_{m\neq n}; (a_n, \eta_n, T_n)\}$, let $(a_n^*, \eta_n^*, \Gamma_n^*)$ be a state of optimal control. Then there exists a $\mathcal{U}_{n,t}$-adapted pair of states $(W_{n,t}, \alpha_{n,t})$ in the*

[6]If process a_n is $\mathbb{G}_n$-adapted, then α_n is $\mathbb{G}_n$-adapted.

L^2 space that satisfies (7) where $a_n = a_n^*$ and $\eta_n = \eta_n^*$. Moreover, for almost all $t \in [0, T_n]$, the optimal control a_n^* and η_n^* are almost certain to coincide:

$$H(a_{n,t}^*, \eta_{n,t}^*, c_{n,t}, \alpha_{n,t}) = \sup_{a_n, \eta_n} H(a_{n,t}, \eta_{n,t}, c_{n,t}, \alpha_{n,t}) \tag{8}$$

The contract (c_n) is considered <u>admissible</u> if and only if (7) yields a sensitivity of the continuation value with respect to project output $\alpha_{n,t}$ greater than the marginal effort cost of the agent λ, i.e., $\alpha_{n,t} \geq \lambda$ for $0 \leq t \leq T$. Moreover, the company's ESG performance is given by $\eta_{n,t} = \frac{1}{2Q}\left(P - Q\sum_{m \neq n} \eta_{m,t} - \theta_n\right)$ for $0 \leq t \leq T$, indicating that the marginal benefit of the agent's action must be greater than or equal to their own marginal cost for the corresponding action.

Oligopoly competition. To obtain an exact analytical solution, we consider the case of a duopoly competitive market, i.e., $N = 2$, without any loss of generality. Our analysis will focus on this scenario. The optimal ESG performance in duopoly competition is summarized in the following proposition:

Proposition 3.4. *With the government's ESG incentive policy (P) and ESG incentive form $\left(P - Q\sum_{m=1}^{N} \eta_{m,t}\right)\eta_{n,t}$ in place, the agent is empowered to make ESG performance decisions. In a market with two companies, i.e., $N = 2$, the optimal ESG performance of the two companies through the agent's decision is:*

$$\begin{cases} \eta_{1,t}^* = \frac{P - 2\theta_1 - \theta_2}{3Q}, \\ \eta_{2,t}^* = \frac{P - \theta_1 - 2\theta_2}{3Q}. \end{cases} \tag{9}$$

Incentive-compatible condition of policy. Given that the economic benefits of a company's ESG performance are not directly reflected in the project's cash flow, but rather the cost associated with ESG performance can reduce the project's cash flow, government policy-maker must provide incentive fiscal policies to encourage companies to improve their ESG performance. Therefore, in our model, a rational agent's current income from ESG performance under incentive policies must be no less than that under non-incentive policies.

Proposition 3.5. *By comparing the parts of the Hamiltonian that are related to ESG performance, we obtain the policy incentive compatibility condition, which takes into account the contract incentive compatibility:*

$$\min_{n=1,2}\{2P - 3\theta_n\} \geqslant 0. \tag{10}$$

Further, we derive the agent's incentive-compatible strategy as follows.

Proposition 3.6. *If contract* (c_n) *is admissible and policy* (P) *satisfies the incentive-compatible condition, the corresponding agent's incentive-compatible strategy* (a_n, η_n, T_n) *is given by* $a_{n,t} = \bar{a}$ *and* $\eta_{n,t}$ *satisfies* (9) *for* $0 \leq t \leq T_n$ *and that the stopping time* T_n *is the first time of the continuation value* W_n *hitting the termination utility, i.e.* $T_n = \inf\{t \geq 0 : W_{n,t} = R_n\}$.

Next, we turn to the principal's strategy and optimal contract design.

3.2. Principal's problem and optimal contract

In general, there are numerous admissible contracts. Let $\mathcal{A}_n$ denote the set of all admissible contracts for company n, $n = 1, ..., N$. As previously assumed, the optimal contract, denoted by (c_n^*), must also be admissible, i.e., $(c_n^*) \in \mathcal{A}_n$. According to Williams (2015) and Yang and Zhang (2023), we can establish a one-to-one correspondence between the contract $(c_n) \in \mathcal{A}_n$ and the controlled process W_n that is controlled by (c_n, α_n). Therefore, the optimal contract problem takes the co-state $W_{n,t}$ as the unique state variable, with the control variables including (c_n, α_n). As a result, the principal must solve the following stochastic dynamic programming problem:

$$\begin{aligned} &V(w) \equiv \sup_{\alpha_n;(c_n)\in\mathcal{A}_n} \mathbb{E}^{a_n,\eta_n}\left\{\int_0^{T_n} e^{-rt}[(1-\tau)\mathrm{d}X_{n,t} - c_{n,t}\mathrm{d}t] + e^{-rT_n}L_n \middle| W_{n,0} = w\right\}, \\ &\text{s.t.}\mathrm{d}W_{n,t} = (\gamma W_{n,t} - c_{n,t} + \lambda\bar{a})\,\mathrm{d}t + \alpha_{n,t}\sigma_n(1-\tau)\mathrm{d}Z_t^{a_n,\eta_n},\ t\in[0,T_n];\ \alpha_{n,t} \geqslant \frac{\lambda}{1-\tau}, \end{aligned} \tag{11}$$

where T_n is the first time of the continuation value process hitting the termination utility R_n according to Proposition 3.6. Therefore, the principal's value function $V(W_{n,t})$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$
\begin{aligned}
rV_n(W_{n,t}) = \sup_{c_{n,t},\alpha_{n,t}} & (1-\tau)\left(\bar{a} + P^2 - 4\theta_n P - 3\theta_n^2 + (\theta_1+\theta_2)(2P - 9\theta_n - 3\theta_{-n})\right) \\
& - c_{n,t} + (\gamma W_{n,t} - c_{n,t} + \lambda \bar{a})\, V_n'(W_{n,t}) + \frac{1}{2}\alpha_{n,t}^2(1-\tau)^2\sigma_n^2 V_n''(W_{n,t}).
\end{aligned}
\tag{12}
$$

for $0 \le t \le T_n$, where the first item in right hand side of equation is the instantaneous expected utility of the principal, the second term is the expected change due to the drift, and the third term is expected change due to the volatility. Then, the optimal compensation rate $c_{n,t}$ and incentive factor $\alpha_{n,t}$ can be obtained by solving the following problems:

$$
c_{n,t}^*(W_{n,t}) \in \arg\max_{c_{n,t}\ge 0} \left\{ -c_{n,t} - c_{n,t}V'(W_{n,t}) \right\},\ \alpha_{n,t}^*(W_{n,t}) \in \arg\max_{\alpha \ge \frac{\lambda}{1-\tau}} \left\{ \frac{1}{2}\alpha_{n,t}^2(1-\tau)^2\sigma_n^2 V_n''(W_{n,t}) \right\}.
$$

There is a threshold $\bar{W}_n > R_n$ that satisfies $V_n'(\bar{W}_n) = -1$. Solving the first-order conditions yields the optimal compensation $c_{n,t}^*(W_{n,t})$, which is given by:

$$
c_{n,t}^*(W_{n,t}) = \begin{cases} 0, & V_n'(W_{n,t}) \geqslant -1; \\ W_{n,t} - \bar{W}_n, & V_n'(W_{n,t}) < -1. \end{cases}
\tag{13}
$$

The optimal incentive factor is $\alpha_{n,t}^*(W_{n,t}) = \lambda/(1-\tau)$ because the value function $V_n(\cdot)$ is a concave function, meaning that $V_n''(W_{n,t}) < 0$. Equation (13) indicates that the principal pays the agent if and only if the promised utility exceeds $\bar{W}_n$. A higher promised utility results in greater compensation paid by the principal. Furthermore, assuming inefficient termination, the minimum incentive factor is chosen, namely $\alpha_{n,t}^*(W_{n,t}) = \lambda/(1-\tau)$. [7]

In the literature, such as DeMarzo and Sannikov (2006), Sannikov (2008), DeMarzo et al. (2012), and DeMarzo and Sannikov (2016), the optimal incentive factor α_n in contract implementation is typically defined as the percentage of internal equity that the agent must hold. However, in our model, it depends solely on the agent's marginal effort cost λ. The promised value's sensitivity to (after-tax) project output, which is equivalent to the marginal

[7] In economic equilibrium, risk should be distributed as much as possible to risk-neutral actors rather than risk-averse ones.

utility, must intuitively cover the agent's marginal effort cost. As argued by DeMarzo and Sannikov (2006), this incentive is costly and should be as small as possible, resulting in $\alpha^*_{n,t}(W_{n,t}) = \lambda/(1-\tau)$ for all times $t \in [0, T_n]$.

Optimal contract. By substituting (13) with $\alpha^*_{n,t}(W_{n,t}) = \lambda/(1-\tau)$ into the HJB equation (12), we obtain the following segmented second-order ordinary differential equation (ODE):

$$\begin{cases} rV_n(x) = (1-\tau)\left(\bar{a} + P^2 - 4\theta_n P - 3\theta_n^2 + (\theta_1+\theta_2)(2P - 9\theta_n - 3\theta_{-n})\right) \\ + \left(\gamma x + \lambda \bar{a}\right) V_n'(x) + \frac{1}{2}\lambda^2\sigma_n^2 V_n''(x),\ x \in [R_n, \bar{W}_n), \\ V_n(x) = V_n(\bar{W}_n) - x + \bar{W}_n,\ x \in [\bar{W}_n, \infty). \end{cases} \tag{14}$$

To determine a solution to this equation and the boundary $\bar{W}_n$, three boundary conditions are necessary. The first boundary condition arises from the agent's obligation to terminate the contract to hold the agent's value to R_n, resulting in $V_n(R_n) = L_n$. The second boundary condition is the conventional "smooth pasting" condition, which states that the first derivatives must match at the boundary, thus $V_n'(\bar{W}_n) = -1$. The final boundary condition is the "super contact" condition for the optimality of $\bar{W}_n$, requiring that the second derivatives match at the boundary, i.e. $V_n''(\bar{W}_n-) = V_n''(\bar{W}_n+) = 0$.

3.3. Government's problem and optimal ESG incentive

Upon solving the competitive market game equilibrium and constructing the optimal contract within the company, the government receives taxes from the enterprise project cash flow at a specific tax rate. As an overall social planner, government policymaker is also affected by the externalities of enterprises. Thus, tax revenue and externalities jointly determine the utility of the government policymaker. In this problem, policymaker aims to maximize their cumulative discount utility by formulating the optimal ESG incentive policy. Let $\mathcal{P}$ denote the set of all ESG incentives. As previously assumed, the optimal ESG incentive satisfies $(P^*) \in \mathcal{P}$. To normalize this problem, government policymaker must solve the following random optimization problems:

$$\begin{aligned}
&\sup_{P\in\mathcal{P}} \mathbb{E}\left\{\int_0^{\min\{T_n\}} e^{-rt}\left(\tau\sum_{n=1}^{N}\mathrm{d}X_{n,t}+\sum_{n=1}^{N}\mathrm{d}Y_{n,t}-\left(P-Q\sum_{n=1}^{N}\eta_{n,t}\right)\sum_{n=1}^{N}\eta_{n,t}\right)\right\},\\
&\text{s.t. } \mathrm{d}X_{n,t}=\left[a_{n,t}+\left(P-Q\sum_{m=1}^{N}\eta_{m,t}\right)\eta_{n,t}-\theta_n\eta_{n,t}\right]\mathrm{d}t+\sigma_n\mathrm{d}Z_t,\\
&\qquad \mathrm{d}Y_{n,t}=\left(-b_n+\eta_{n,t}\right)\mathrm{d}t+\phi_n\mathrm{d}B_t.
\end{aligned} \tag{15}$$

Since the problem (15) does not involve intertemporal choice of government policymaker, it can be solved using static programming. To ensure individual rationality of government policymaker, an effective ESG incentive must ensure that the utility of government policymaker is no less than when they do not implement ESG incentive. Therefore, an effective ESG incentive must satisfy $\left[1+(\tau-1)\left(P-Q\sum_{n=1}^{N}\eta_{n,t}\right)\right]\sum_{n=1}^{N}\eta_{n,t}\geqslant 0$, i.e.

$$P\leqslant\frac{1}{1-\tau}+Q\sum_{n=1}^{N}\eta_{n,t}. \tag{16}$$

As an example, we consider the duopoly competition market, i.e., $N=2$. Solving problem (15) yields the optimal ESG incentive factor P^*, which is given by

$$P^*=\frac{6+3(\theta_1+\theta_2)(3-2\tau)}{4(1-\tau)}. \tag{17}$$

The increase in the cash flow drift term that enterprise n can obtain from ESG performance is denoted as $\Pi_{n,t}$ and can be calculated as follows:

$$\Pi_{n,t}=\frac{\left[2+(3-2\tau)\theta_n+(7-6\tau)\theta_{-n}\right]\left[6+(1+2\tau)\theta_n+(5-2\tau)\theta_{-n}\right]}{48Q(1-\tau)^2}. \tag{18}$$

The aggregate ESG performance of firms in a duopoly competitive market under the optimal ESG incentive policy is given by:

$$\eta^*_{1,t}+\eta^*_{2,t}=\frac{2+\theta_1+\theta_2}{2Q(1-\tau)}. \tag{19}$$

To sum up, we have the following proposition.

Proposition 3.7. ***(Optimal contract and optimal ESG incentive)*** *Suppose that the project's cash flow is defined by (1), and the externality is defined by (2). The optimal compensation $c^*_{n,t}(W_{n,t})$ offered by the principal that maximizes the agent's value while delivering the value $W_{n,0} = w \geq R$ to the agent takes the following form:*

$$c^*_{n,t}(W_{n,t}) = \begin{cases} 0, & V_n'(W_{n,t}) \geqslant -1; \\ W_{n,t} - \bar{W}_n, & V_n'(W_{n,t}) < -1. \end{cases}$$

Government policymaker maximizes his own utility and encourage enterprises to actively improve ESG performance by providing ESG performance incentives $\left(P - Q\sum_{n=1}^{N} \eta_{n,t}\right)$. Under such an ESG incentive scheme, the optimal ESG incentive factor P^* takes the following form:

$$P^* = \frac{6 + 3(\theta_1 + \theta_2)(3 - 2\tau)}{4(1 - \tau)}.$$

If $W_{n,t} \in [R_n, \bar{W}_n)$, the agent's continuation value $W_{n,t}$ evolves according to

$$\mathrm{d}W_{n,t} = (\gamma W_{n,t} + \lambda \bar{a})\, \mathrm{d}t + \lambda \sigma_n \mathrm{d}Z_t^{a_n,\eta_n}.$$

The principal's value $V_n(W_{n,t})$ at any time $0 \leq t \leq T_n$ is the function of the current continuation value $W_{n,t}$, where the function $V_n(\cdot)$ is a solution of the following ODE:

$$\begin{aligned} rV_n(x) =& (1 - \tau)\left(\bar{a} + P^{*2} - 4\theta_n P^* - 3\theta_n^2 + (\theta_1 + \theta_2)(2P^* - 9\theta_n - 3\theta_{-n})\right) \\ &+ (\gamma x + \lambda \bar{a})\, V_n'(x) + \frac{1}{2}\lambda^2 \sigma_n^2 V_n''(x), \end{aligned}$$

and the boundary conditions are $V_n(R_n) = L_n$, $V_n'(\bar{W}_n) = -1$, and $V_n''(\bar{W}_n-) = 0$.

Moreover, if $W_{n,t} \geq \bar{W}_n$, the manager's continuation value $W_{n,t}$ evolves according to

$$\mathrm{d}W_{n,t} = \left(\gamma W_{n,t} - W_{n,t} + \bar{W}_n + \lambda \bar{a}\right) \mathrm{d}t + \lambda \sigma_n \mathrm{d}Z_t^{a_n,\eta_n}.$$

The principal's value $V_n(W_{n,t})$ at any time $0 \leq t \leq T_n$ is the function of the current contin-

uation value $W_{n,t}$, where the function $V_n(\cdot)$ is a solution of the following ODE:

$$V_n(x) = V_n(\bar{W}_n) - x + \bar{W}_n.$$

The optimal contract (c_n^*) mentioned above is explicit and easy to compute if the principal's value function $V_n(\cdot)$ is known. Fortunately, the value function is a solution to the ODE, which can be easily solved numerically. Figure 1 provides an example of the principal's value function. As expected, the figure indicates that the principal's value initially increases with the promised value W_n and then decreases. This phenomenon arises from two opposing forces as the promised value increases: one increases the value since the probability of inefficient termination decreases, and the other decreases it for an apparent reason. The promised value $\bar{W}_n$ of the agent represents the break-even point of the two forces, as shown in the figure.

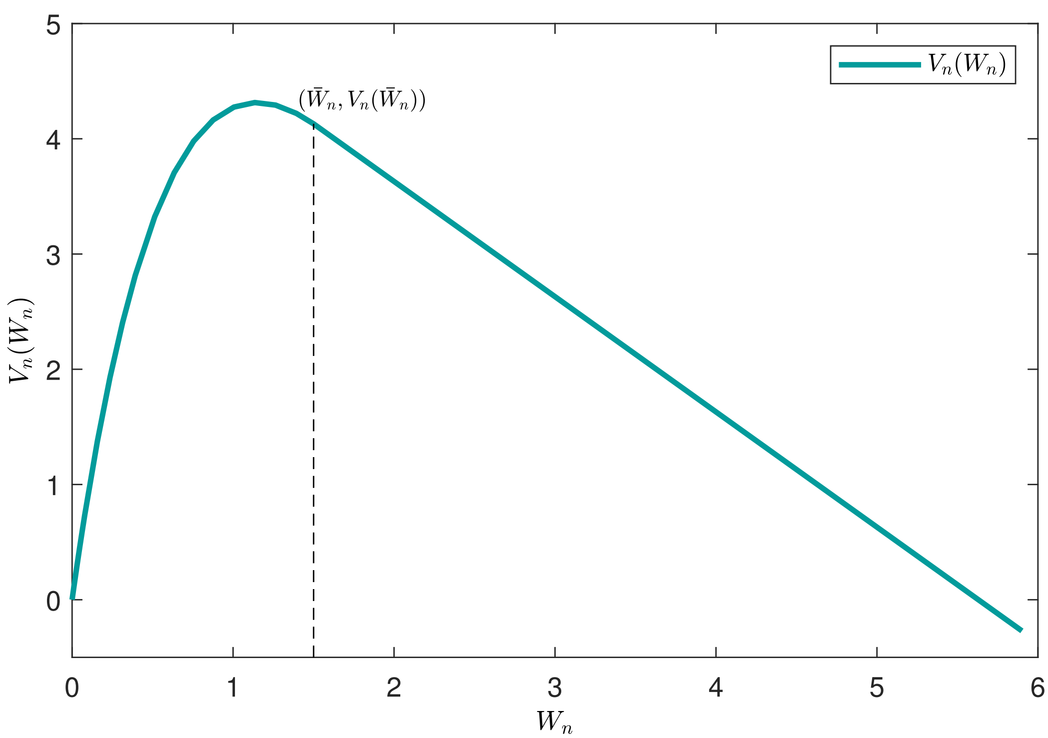


Figure 1. The principal's value function $V_n(W_n)$ given by (14) versus the agent's promised value starting from (R_n, L_n), where $\bar{a} = 10$, $r = 4.6\%$, $\gamma = 5\%$, $\sigma_n = 5$, $b_n = 2$, $\lambda = 60\%$, $\tau = 25\%$, $\theta_n = 40\%$, $Q = 1$, $R = 0$, and $L = 0$.

4. Optimal contract and ESG incentives under common ownership

Most existing studies on the ESG performance of companies are based on the implicit assumption that the behaviors of companies in an investor's portfolio are independent of each other and do not interfere with one another. Therefore, the ESG performance of each company is not affected by the behaviors of other firms in the portfolio. However, in reality, the existence of institutional investors may affect the relevant behaviors of enterprises through common ownership. In this section, we relax the assumption that the behaviors of firms in investors' portfolios are independent of each other and explicitly discuss the impact of institutional investors' common ownership on firms' ESG performance.

4.1. Collusive fraud effect

Firstly, we examine the collusive fraud impact of joint shareholding. Institutional investors who possess common ownership can encourage collusion among firms. The objective of mutual institutional investors is not solely to maximize the profits of a single enterprise, but to enhance the value of the portfolio. Consequently, they are motivated to prompt firms to curtail excessive competitive strategies and prevent mutual harm resulting from competition. To differentiate between various scenarios, this section employs the superscript "c" instead of the superscript "$*$" to denote the resolution of the economic optimization problem in the conspiratorial fraud instance.

Without loss of generality, we assume that institutional investors own shares of all companies in the industry. In the case of common ownership, look at the companies in the industry as a whole. We use the total ESG performance of companies in the industry Φ_t as an alternative control variable, i.e

$$\Phi_t = \sum_{n=1}^{N} \eta_{n,t}. \tag{20}$$

We can obtain the industry's total operating cash flow X by modifying (1) as follows:

$$\mathrm{d}X_t = \left[\sum_{n=1}^{N} a_{n,t} + (P - Q\Phi_t)\,\Phi_t - \frac{1}{n}\Phi_t \sum_{n=1}^{N} \theta_n \right] \mathrm{d}t + \sigma \mathrm{d}Z_t,\ \ X_0 = x, \tag{21}$$

where $\frac{1}{n}\sum_{n=1}^{N} \theta_n$ represents the marginal cost of ESG performance as measured by the equally-weighted marginal cost of all companies in the industry, Z_t is a Brownian motion composed of all $Z_{n,t}$, and σ is the corresponding total risk. Likewise, we can determine the total externality process Y by modifying (2) as follows:

$$\mathrm{d}Y_t = \left(-\sum_{n=1}^{N} \sigma_n b_n + \Phi_t \right) \mathrm{d}t + \phi \mathrm{d}B_t,\ Y_0 = y. \tag{22}$$

where B_t is a Brownian motion composed of all $B_{n,t}$, and ϕ is the corresponding total risk. Initially, let us examine the problem faced by the agent. In a similar fashion, we employ the stochastic maximum principle proposed by Bismut (1973) to establish the necessary condition for optimality. Furthermore, Lemma 4.1 assists us in constructing the co-state:

Lemma 4.1. *The optimal control is attained by maximizing the Hamiltonian, and the co-state's evolution is governed by the Hamiltonian's differential. Given a strategic profile* $\{(\sum_{n=1}^{N} c_n); (P); (\sum_{n=1}^{N} a_n, \Phi_t, T)\}$, *we can obtain the co-state* W_t *by solving the following BSDE:*

$$\begin{aligned} \mathrm{d}W_t &= \left(\gamma W_t - \sum_{n=1}^{N} c_{n,t} + \lambda \sum_{n=1}^{N} a_{n,t} \right) \mathrm{d}t + (1-\tau)\alpha_t \sum_{n=1}^{N} \sigma_n \mathrm{d}Z_t^{\sum_{n=1}^{N} a_n, \sum_{n=1}^{N} \eta_n}, \\ W_T &= \sum_{n=1}^{N} R_n. \end{aligned} \tag{23}$$

To make a contract admissible, the principal must provide the agent with sufficient incentives relative to the effort cost. Formally, we have

Proposition 4.2. *Given a strategy profile* $\{(\sum_{n=1}^{N} c_n); (P); (\sum_{n=1}^{N} a_n, \Phi_t, T)\}$, *we define* $(\sum_{n=1}^{N} a_n^c, \Phi^c, \Gamma^c)$ *as the optimal control pair state. Then, there exists a pair of* $\mathcal{U}_t$*-adapted states* $(W_t, \sum_{n=1}^{N} \alpha_{n,t})$ *in the* L^2 *space that satisfy (7), where* $a_n = a_n^c$ *and* $\Phi = \Phi^c$, *and for almost every* $t \in [0,T]$, *the optimal controls* a_n^c *and* Φ^c *are almost certain to coincide:*

$$H\left(\sum_{n=1}^{N} a_{n,t}^c, \Phi_t^c, \sum_{n=1}^{N} c_{n,t}, \alpha_t \right) = \sup_{\sum_{n=1}^{N} a_n, \Phi} H\left(\sum_{n=1}^{N} a_{n,t}, \Phi_t, \sum_{n=1}^{N} c_{n,t}, \alpha_t \right) \tag{24}$$

The contract $(\sum_{n=1}^{N} c_n)$ *is deemed admissible if and only if (23) yields the sensitivity of the continuation value concerning the project output* α_t *greater than the agent's marginal* *effort cost λ, i.e., $\alpha_t \geq \lambda$ for $0 \leq t \leq T$. The company's ESG performance is given by $\Phi_t = \frac{1}{2NQ}\left(2P - \sum_{n=1}^{N} \theta_n\right)$ for $0 \leq t \leq T$, indicating that the agent's marginal benefit of action must surpass (or be equal to) the corresponding action's marginal cost.*

In this subsection, we maintain the assumption of a duopoly market for our ultimate solution, i.e., $N = 2$. Given a strategy profile $\{(\sum_{n=1}^{N} c_n); (P)\}$, the total ESG performance of companies with common ownership is given by:

$$\Phi_t = \frac{2P - \theta_1 - \theta_2}{4Q}. \tag{25}$$

By comparing the total ESG performance of common ownership (25) with the ESG performance in a competitive market $\eta_{1,t} + \eta_{2,t}$ described in Proposition 3.4, we can analyze the market's total ESG performance change caused by common ownership for a given strategy profile $\{(\sum_{n=1}^{N} c_n); (P)\}$. If $9(\theta_1 + \theta_2) - 2P \geqslant 0$, common ownership leads to improved overall ESG performance in the market due to the collusive fraud effect. Conversely, if $9(\theta_1 + \theta_2) - 2P < 0$, common ownership results in lower overall ESG performance in the market because of the collusive fraud effect.

By following a familiar calculation process, we obtain the following ordinary differential equation satisfied by the principal's value function:

$$\begin{cases} rV(x) = (1-\tau)\left(N\bar{a} + (P - Q\Phi_t)\,\Phi_t - \frac{1}{n}\Phi_t \sum_{n=1}^{N} \theta_n\right) \\ + (\gamma x + \lambda N\bar{a})\, V'(x) + \frac{1}{2}\lambda^2 (\sum_{n=1}^{N} \sigma_n)^2 V''(x),\ x \in [R, \tilde{W}), \\ V(x) = V(\tilde{W}) - x + \tilde{W},\ x \in [\tilde{W}, \infty). \end{cases} \tag{26}$$

where $\tilde{W}$ is the threshold $\tilde{W} > R$ such that $V'(\tilde{W}) = -1$. It is evident that the principal's payment to the agent is contingent on the promised value exceeding $\tilde{W}$. The compensation rate increases with a higher promised value. There are three boundary conditions: (1) $V(\sum_{n=1}^{N} R_n) = \sum_{n=1}^{N} L_n$, (2) $V'(\tilde{W}) = -1$, and (3) $V''(\tilde{W}-) = V''(\tilde{W}+) = 0$. Figure 2(a) illustrates a numerical example of the principal's value function.

The following question pertains to government policymakers. Analogous to (15), policymaker must solve the subsequent stochastic optimization problem:

$$\begin{aligned}
&\sup_{P\in\mathcal{P}} \mathbb{E}\left\{\int_0^T e^{-rt}\left(\tau \mathrm{d}X_t + \mathrm{d}Y_t - (P - Q\Phi_t)\,\Phi_t\right)\right\}, \\
&\text{s.t. } \mathrm{d}X_t = \left[\sum_{n=1}^N a_{n,t} + (P - Q\Phi_t)\,\Phi_t - \frac{1}{n}\Phi_t \sum_{n=1}^N \theta_n\right]\mathrm{d}t + \sigma \mathrm{d}Z_t, \\
&\qquad \mathrm{d}Y_t = \left(-\sum_{n=1}^N \sigma_n b_n + \Phi_t\right)\mathrm{d}t + \phi \mathrm{d}B_t.
\end{aligned} \tag{27}$$

In a duopoly competition market, i.e., $N = 2$, solving problem (27) yields the optimal ESG incentive factor P^c, which is expressed as:

$$P^c = \frac{2 - \tau(\theta_1 + \theta_2)}{2(1-\tau)}. \tag{28}$$

For the optimal strategy profile $\{(\sum_{n=1}^N c_n^c); (P^c); (\sum_{n=1}^N a_n^c, \Phi_t^c, T^c)\}$, we compare the total ESG performance of common ownership Φ_t^c with the ESG performance in a competitive market $\eta_{1,t}^* + \eta_{2,t}^*$ described in (19). This yields the following inequality:

$$\Phi_t^c = \frac{2 - (\theta_1 + \theta_2)}{4Q(1-\tau)} \leqslant \frac{2 + \theta_1 + \theta_2}{2Q(1-\tau)} = \eta_{1,t}^* + \eta_{2,t}^*. \tag{29}$$

The findings indicate that the common ownership of institutional investors reduces the overall ESG performance of the market due to the collusive fraud effect when government policy-makers formulate the optimal ESG incentive policy (P^c). This differs from the conclusion when ESG incentive factor (P) are fixed.

4.2. Synergistic governance effect

In this section, we utilize the superscript "s" instead of "$*$" to differentiate the solution of the economic optimization problem from the synergistic governance scenario. Institutional investors, who serve as a crucial link between different enterprises, can facilitate the flow of information and significantly reduce information asymmetry through their common ownership, thereby promoting enterprise cooperation.

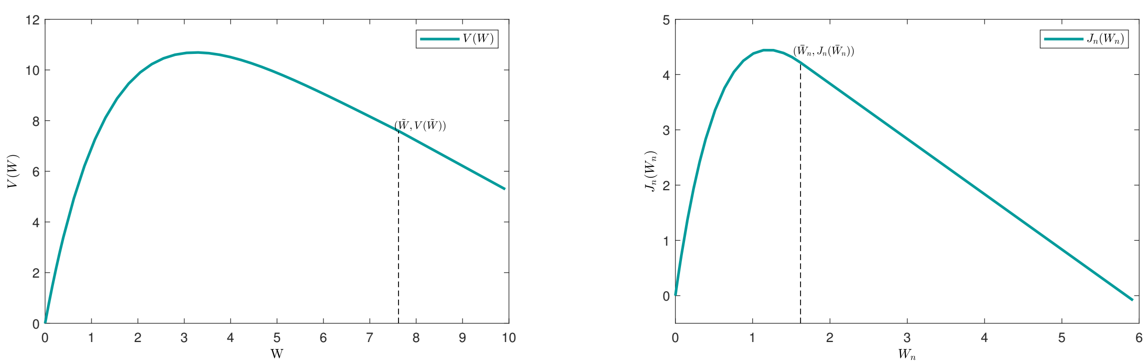
In the context of common ownership, we assume that companies within an industry can engage in cooperative efforts to decrease ESG performance costs through the collaborative governance effect. Let us suppose that as a result of such collaboration, the ESG performance cost of company n, where $n = 1, ..., N$, decreases to $\underline{\theta}$. Here, $\underline{\theta}$ satisfies the following relationship:

$$\underline{\theta} \leqslant \min\{\theta_1, ..., \theta_N\}. \tag{30}$$

After performing a standard calculation, we obtain the following ODE, which is satisfied by the principal's value function of company n, where $n = 1, ..., N$:

$$\begin{cases} rJ_n(x) = (1-\tau)\left(\bar{a} + P^2 - 4\underline{\theta}P - 3\underline{\theta}^2 + 4\underline{\theta}(P - 6\underline{\theta})\right) \\ + (\gamma x + \lambda\bar{a})\, J_n'(x) + \frac{1}{2}\lambda^2\sigma_n^2 J_n''(x),\ x \in [R, \hat{W}_n), \\ J_n(x) = J_n(\hat{W}_n) - x + \hat{W}_n,\ x \in [\hat{W}_n, \infty). \end{cases} \tag{31}$$

where $\hat{W}_n$ is the threshold $\hat{W}_n > R$ such that $J_n'(\hat{W}_n) = -1$. Clearly, the principal's payment to the agent is made if and only if the promised value is greater than $\hat{W}_n$. A higher promised value corresponds to a higher compensation rate. The principal's value function is subject to three boundary conditions: (1) $J_n(R_n) = L_n$, (2) $J_n'(\hat{W}_n) = -1$, and (3) $J_n''(\hat{W}_n-) = 0$. Figure 2(b) presents a numerical example of the principal's value function.



(a) $V(W)$ vs. W

(b) $J_n(W)$ vs. W

Figure 2. The principal's value function (a) $V(W)$ given by (26) versus the agent's promised value starting from $(\sum_{n=1}^{N} R_n, \sum_{n=1}^{N} L_n)$ and (b) $J_n(W_n)$ given by (31) versus the agent's promised value starting from (R_n, L_n), where $\bar{a} = 10$, $r = 4.6\%$, $\gamma = 5\%$, $\sigma_n = 5$, $b_n = 2$, $\lambda = 60\%$, $\tau = 25\%$, $\theta_n = 40\%$, $Q = 1$, $N = 2$, $R = 0$, and $L = 0$.

Fix incentive factor situation. In this subsection, we continue to adopt the assumption of a duopoly market as our final solution, i.e., $N = 2$. Specifically, we do not take into account the optimization decisions of government policymakers. By comparing the total ESG performance of common ownership, denoted as $\eta^s_{1,t} + \eta^s_{2,t}$, with the ESG performance in a competitive market, represented by $\eta_{1,t} + \eta_{2,t}$ as described in proposition 3.4, we can analyze the change in total ESG performance of the market caused by common ownership for a given strategy profile, where $n = 1, 2$ and $\{(c_n); (P)\}$ is the strategy profile. The following inequality holds:

$$\eta^s_{1,t} + \eta^s_{2,t} = \frac{2P - 6\underline{\theta}}{3Q} \geqslant \frac{2P - 3\theta_1 - 3\theta_2}{3Q} = \eta_{1,t} + \eta_{2,t}. \tag{32}$$

For a given strategy profile $\{(c_n); (P)\}$, $n = 1, 2$, we find that common ownership leads to improved overall ESG performance in the market because of the synergistic governance effect.

Optimal incentive factor situation. Next, we consider the optimization decisions of government policymakers. For the optimal strategy profile $\{(c^s_n); (P^s); (a^s_{n,t}, \eta^s_{n,t}, T^s)\}$, where $n = 1, 2$, we compare the total ESG performance of common ownership, denoted as $\eta^s_{1,t} + \eta^s_{2,t}$, with the ESG performance in a competitive market, represented by $\eta^*_{1,t} + \eta^*_{2,t}$ as described in (19). The following inequality holds:

$$\eta^s_{1,t} + \eta^s_{2,t} = \frac{2 + 2\underline{\theta}}{2Q(1 - \tau)} \leqslant \frac{2 + \theta_1 + \theta_2}{2Q(1 - \tau)} = \eta^*_{1,t} + \eta^*_{2,t}. \tag{33}$$

The findings indicate that the common ownership of institutional investors leads to a decrease in the total ESG performance of the market due to the synergistic governance effect when government policymaker devises the optimal ESG incentive policy (P^s). This conclusion differs from that reached when ESG incentive factor (P) are fixed.

5. Security implementation of optimal contracts

In the preceding section, we concluded the derivation of the optimal contract. Nonetheless, the optimal contract is abstract and, thus, challenging to execute directly. Therefore, we propose using a portfolio of securities, or financial instruments, to implement the optimal contract. If the cash flows produced by the project align perfectly with the claims on these securities, both the agent and the principal will willingly partake in the project, and the agent's incentive compatibility constraint will be established, indicating that the optimal contract is implemented by these securities.

5.1. Security implementation

Drawing on DeMarzo and Sannikov (2006), we propose an approach for realizing the optimal contract using a combination of equity, long-term bonds, and a credit line. It should be noted that other methods of implementation are also possible. In Appendix A, we present a demonstration of the optimal contract implementation discussed in this section. Throughout this section, we maintain our assumption of a duopolistic competitive market. Initially, we consider the perspective of government decision-makers external to the firm. Policymakers need only provide ESG incentives, based on (17) and $\left(P - Q\sum_{n=1}^{N}\eta_{n,t}\right)$, to all firms operating in a competitive market. In response, rational entrepreneurs will spontaneously enhance their ESG performance.

We now turn to the implementation of the optimal contract within each company, denoted by $n = 1, 2$. (1) **Equity allocation.** Firstly, we consider equity allocation. The investor assigns α_n of internal equity to the manager, while retaining the remaining $1 - \alpha_n$ of equity. The distribution of equity aligns with the incentive compatibility conditions and optimal incentive factors outlined in the optimal contract discussed earlier. The α_n internal equity eliminates the manager's incentive to reduce workload as a means of minimizing effort costs. As a result, the manager can offset the maximum work cost by paying dividends.

(2) **Credit line.** Secondly, we consider the credit line. Let us assume the existence of a liquid credit market (or short-term bond market) that provides Company n (where $n = 1, 2$) with a credit line (or short-term bond issuance line) of up to S_n. The interest rate on the credit line withdrawal balance $M_{n,t}$ is fixed at r. The manager is responsible for borrowing and repaying funds on behalf of the company. The current draw balance on the credit line $M_{n,t}$ corresponds to the manager's continuation value in the optimal contract, and the relationship between them is as follows:

$$W_{n,t} = R_n + \alpha_n(S_n - M_{n,t}). \tag{34}$$

If the current draw balance on the credit line, $M_{n,t}$, exceeds S_n, the company will default and the project will come to an end. In the event that the credit line draw balance is entirely repaid, any remaining cash held by the company will be used to issue dividends. It is worth noting that, as per (34), $M_{n,t} = 0$ corresponds to the dividend payment threshold. Therefore, we can deduce that the credit line, S_n, corresponds to the reward payment threshold, $\bar{W}_n$, in the optimal contract, and the corresponding relationship is:

$$S_n = \frac{1}{\alpha_n}\left(\bar{W}_n - R\right). \tag{35}$$

The optimal contract's historical dependence is established using the credit line, whereby the draw balance $M_{n,t}$ acts as a ledger that collectively monitors the continuation value. The credit line can be viewed as a company's cash flow reservoir, and when the water level reaches its maximum capacity, the company will become bankrupt.

(3) **Long-term bonds.** Thirdly, we consider long-term bonds. The company will issue long-term bonds that pay continuous coupons at a debt rate of k. Without loss of generality, let the coupon rate be r, such that the face value of the debt is $D_n = k/r$. The face value of the long-term bonds issued by the company will be determined based on the sustainable value of the project cash flows and the credit line:

$$D_n = \frac{\bar{a} + \Pi_n}{r} - \frac{\gamma R_n}{r\alpha_n} - \frac{\gamma S_n}{r}. \tag{36}$$

The first term on the right-hand side of the equation represents the value of the cash flow, the second term represents the value of all equity upon project termination, and the third term represents the credit line. Therefore, (36) represents the total value of the company at the time of project termination as the sum of the market value of equity, the market value of long-term bonds, and the market value of the credit line (short-term bonds). In fact, this equation holds true at any point in time. As per Demarzo et al. (2006), the purpose of issuing long-term bonds is to ensure that the capital structure is incentive-compatible, meaning that the entrepreneur's effort level is maximized, and dividends are paid only if the credit is repaid, even in the case of early contract termination, which prevents the entrepreneur from overdrawing the credit maliciously.

5.2. Application example

On May 31, 2022, the State Administration of Taxation (SAT) released the "Guidelines on Tax and Fee Preferential Policies to Support Green Development" [8]. The guidelines outline 56 tax and fee preferential policies that the state has implemented to support green development across four areas: supporting environmental protection, promoting energy conservation and environmental protection, encouraging the comprehensive utilization of resources, and promoting the development of low-carbon industries. These policies include a reduction or exemption of enterprise income tax on income generated from qualified environmental protection activities, as well as an enterprise income tax credit for investments in specialized environmental protection equipment. In this subsection, we refer to this policy and potential business scenarios to provide examples of policy formulation and efficient implementation.

Consider two competing companies operating in the industry. The entrepreneur's maximum effort level can increase the company's cash flow by an average of $\bar{a} = 2$ million per year, with a marginal cost of $\lambda = 60\%$. Additionally, the company's production and operation activities result in carbon dioxide emissions that have a negative impact on the environment. To standardize the dimensions, we use dollars as the unit of measurement for the impact of carbon dioxide emissions on the environment, and the average economic value of the cumulative environmental damage caused by carbon dioxide emissions is $b = 2$ million per year. Managers have the option to treat exhaust gas before discharge to reduce the externalities resulting from the company's operations. This is a means of improving ESG performance, denoted by $\eta_{1,t}$ and $\eta_{2,t}$. The marginal cost of waste gas treatment for both companies is $\theta_1 = \theta_2 = 40\%$.

Externally, the government can encourage entrepreneurs to actively participate in waste gas treatment by offering ESG incentive fiscal policies. In our model, we suggest that government policymakers take a competitive approach to fiscal policy incentives. When neither company has achieved ESG performance, as per (17), the government rewards each company $P^* = 4$ for every unit of ESG performance. As both companies progressively enhance their ESG performance, the incentives provided by the government are determined by the

[8]See, http://www.chinatax.gov.cn/chinatax/n810341/n810825/c101434/c5175740/content.html

scale of $\left(P - Q\sum_{n=1}^{N}\eta_{n,t}\right)$. Following this policy, both companies will improve their ESG performance to $\eta_{1,t}^{*} = \eta_{2,t}^{*} = 0.93$ million per year.

Within the company, investors must incentivize managers through contractual agreements to maximize efforts towards enhancing the company's cash flow. Our model suggests that optimal contracts can be achieved through the issuance of internal equity, long-term bonds, and credit lines:

Table 1. Optimal securities issuance for a typical scenari

Securities issuance	Symbol	Percentage/Amount
Internal equity	α	80.00%
Long-term bonds	D	33.504
Credit line	S	1.875

To motivate the manager to exert the highest level of effort, they must hold 80.00% of the internal equity, allowing the entrepreneur to compensate for the private cost of the manager's efforts with profits generated from changes in the company's cash flow. Additionally, the company should issue long-term debt of 33.504 billion to meet its operating capital needs, and the repayment of long-term bond interest should be used to adjust the company's profits. The financial market can provide the company with a credit line of 1.875 billion, which allows the company to borrow short-term money within the limit at any time. The balance of short-term borrowing can be used as a ledger for investors to track the entrepreneur's commitment value. If the company performs well and repays its short-term loans in full, the remaining cash flow can be used for dividend payments, and the manager can receive the corresponding dividend based on the 80.00% internal equity ratio. However, if the company performs poorly and is unable to repay its debts, and the balance of short-term borrowings exceeds the credit line provided by the financial market, the company will fail.

6. Numerical analysis and empirical implications

We now turn to the model implications including the analysis of value functions, ESG incentive and ESG performance problems by numerical tests. We wonder how they are affected by common ownership, and how they are related to model parameters.

The primary model utilized in this study is based on the model presented in Section 3, which focuses on optimal contracts in an oligopolistic competitive market. To conduct the numerical analysis, we selected model parameters that are as representative as possible, with reference to previous studies by DeMarzo and Sannikov (2006), DeMarzo et al. (2012), and Yang and Zhang (2023). Unless otherwise stated, we used the annualized baseline parameter values specified in Table 2.

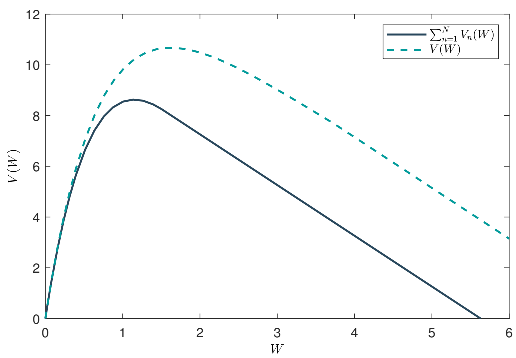
Table 2. Parameters selection

Model parameters	Symbol	Value
Risk-free rate	r	4.6%
Agent's upper bound effort	$\bar{a}$	10
Project cash flow volatility	σ	5
Absolute value of the cumulative rate of project externalities	b	2
Subjective discount rate of professional managers	γ	5%
Marginal effort cost	λ	60%
Basic tax rate	τ	25%
Marginal cost of ESG performance	θ_n	40%
Competition coefficient	Q	1
Number of competing companies	N	2
Outside option value of agents	R	0
Liquidation value of projects	L	0

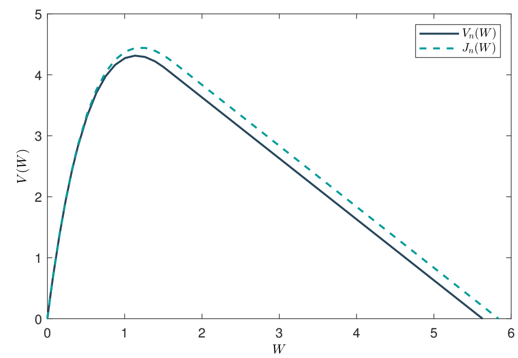
6.1. Value function

Figure 3(a) illustrates the numerical solution for the principal's value in a competitive market and common ownership scenario, which generates a collusive fraud effect. The figure shows that the value function $V(W)$ is higher when common ownership results in collusion among firms, compared to the value function $\sum_{n=1}^{N} V_n(W_n)$ in a competitive market. Economic intuition suggests that collusion among firms, due to common ownership, decreases competition costs, leading to an overall enhancement in the value function.

Figure 3(b) depicts the numerical solution of the principal's value in a competitive market or common ownership scenario, resulting in a synergistic governance effect. The figure shows that the value function $J_n(W_n)$ is higher when common ownership leads to a synergistic governance effect, compared to the value function $V_n(W_n)$ in a competitive market. Economic intuition suggests that common ownership enables synergistic governance among different firms, thereby decreasing the cost of ESG performance and enhancing the value function for investors.



(a) $V_n(W)$ and $V(W)$ vs. W



(b) $V_n(W)$ and $J_n(W)$ vs. W

Figure 3. Model comparison: The figure plots (a) the principal's value in competitive market or common ownership which leads to collusive fraud effect, and (b) the principal's value in competitive market or common ownership which leads to synergistic governance effect, respectively.

Based on the aforementioned results, we conclude that common ownership can enhance the value function of investors, either through collusive fraud or synergistic governance. Common institutional shareholding can positively impact the optimization of corporate governance structures, which, in turn, contributes to the long-term value and ESG performance improvement of enterprises (Ferrell et al., 2016). The findings of these empirical studies align with the conclusions of our theoretical model.

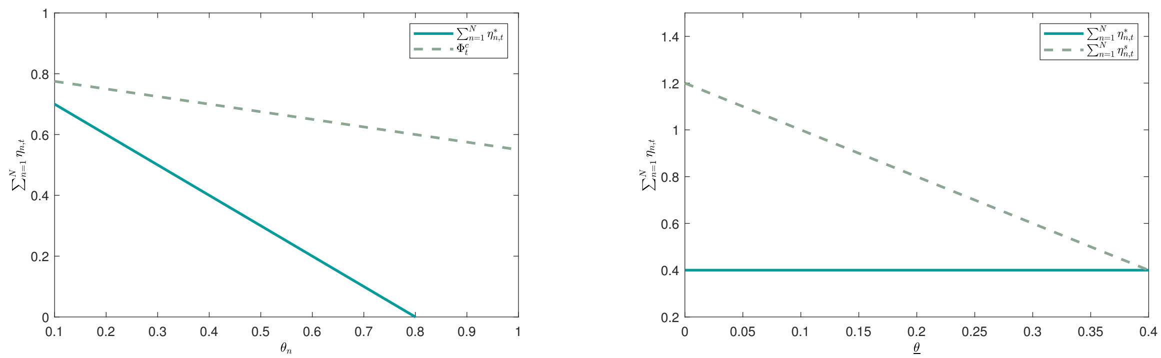
One important topic in ESG research is how a company's ESG characteristics impact executive compensation. Gillan et al. (2010) discovered that CEOs of companies with better ESG performance receive lower compensation. Similarly, Jian and Lee (2015) found a negative correlation between ESG performance and CEO compensation. Ferrell et al. (2016) also found a negative correlation between the measurement standard of CEO excess compensation and ESG performance. However, there is no existing research that concurrently examines common shareholding, ESG, and executive compensation. Our model predicts that common ownership is a crucial determinant of a company's managerial compensation under the government's ESG incentive policies. Therefore, we suggest the following empirical guidelines:

$$\begin{aligned}\text{managerial compensation}_{j,t} =& \kappa_0 + \kappa_1 \text{common ownership}_{j,t} + \kappa_2 \text{control values}_{j,t} \\ &+ \text{Fixed Effect} + \epsilon_{j,t}.\end{aligned} \tag{37}$$

6.2. ESG performance in fix incentive factor scenario

We begin by examining firm ESG performance when government policymakers set the ESG incentive factor at a fixed value P. Figure 4(a) illustrates the numerical solution of the total ESG performance in a competitive market or common ownership scenario, resulting in a collusive fraud effect. Based on our parameter selection, the figure shows that the total ESG performance Φ_t is consistently higher than the total ESG performance $\sum_{n=1}^{N} \eta_{n,t}^{*}$ in a competitive market when common ownership results in a collusive fraud effect. As the marginal cost of a firm's ESG performance increases, the total ESG performance decreases both in competitive markets and under common ownership, which is intuitive from an economic perspective. Moreover, as the marginal cost of ESG performance increases, the degree of improvement in the total ESG performance $\Phi_t - \sum_{n=1}^{N} \eta_{n,t}^{*}$ caused by the collusive fraud effect of common ownership also increases.

Figure 4(b) displays the numerical solution of the total ESG performance in a competitive market or common ownership scenario, resulting in a synergistic governance effect. Based on our parameter selection, the figure shows that the total ESG performance $\sum_{n=1}^{N} \eta_{n,t}^{s}$ is consistently higher than the total ESG performance $\sum_{n=1}^{N} \eta_{n,t}^{*}$ in a competitive market when common ownership results in a synergistic governance effect. As the synergistic ESG marginal cost increases, the total ESG performance decreases in common ownership. When the synergistic ESG marginal cost reaches the ESG performance cost in a competitive market, common ownership no longer improves the total ESG performance. This is intuitive from an economic perspective.



(a) $\sum_{n=1}^{N}\eta_{n,t}^{*}$ and Φ^c vs. θ_n

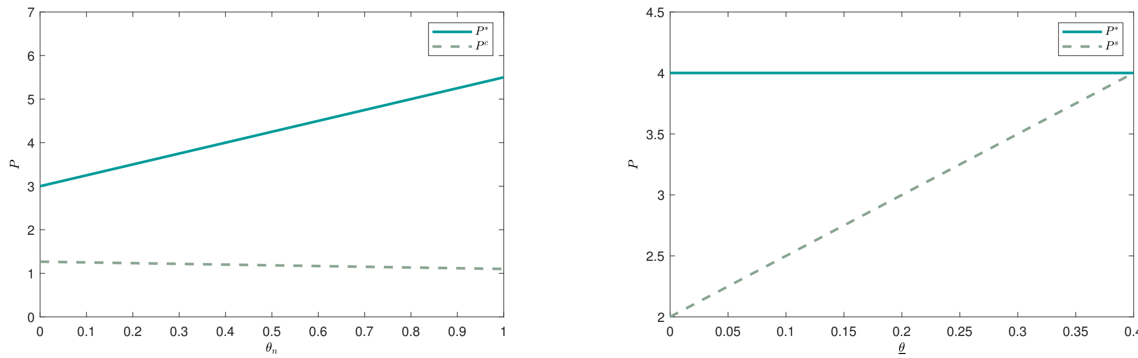
(b) $\sum_{n=1}^{N}\eta_{n,t}^{*}$ and $\sum_{n=1}^{N}\eta_{n,t}^{s}$ vs. $\underline{\theta}$

Figure 4. Model comparison: (a)Collusive fraud effect: Total ESG performance $\sum_{n=1}^{N}\eta_{n,t}^{*}$ and Φ^c varies with the margin cost of ESG performance θ_n. (b) Synergistic governance effect: Total ESG performance $\sum_{n=1}^{N}\eta_{n,t}^{*}$ and $\sum_{n=1}^{N}\eta_{n,t}^{s}$ varies with the margin cost of ESG performance $\underline{\theta}$.

6.3. ESG performance in optimal incentive factor scenario

We then examine firm ESG performance when government policymakers select the optimal ESG incentive factor P^* (or P^c, P^s). Figure 5(a) illustrates the numerical solution of the optimal ESG incentive factor in a competitive market or common ownership scenario, resulting in a collusive fraud effect. Based on our parameter selection, the figure shows that the optimal ESG incentive factor P^* in a competitive market is consistently higher than the value function P^c when common ownership results in a collusive fraud effect. As the marginal cost of a firm's ESG performance increases, the optimal ESG incentive factor P^* increases, while the optimal ESG incentive factor P^c slightly decreases. This implies that in a competitive market, government policymakers must offer more incentives to encourage firms to actively improve ESG performance as the marginal cost of a firm's ESG performance increases. However, in the case of collusion, this phenomenon does not exist.

Figure 5(b) displays the numerical solution of the optimal ESG incentive factor in a competitive market or common ownership scenario, resulting in a synergistic governance effect. Based on our parameter selection, the figure shows that the optimal ESG incentive factor P^* in a competitive market is consistently higher than the value function P^s when common ownership results in a synergistic governance effect. As the synergistic ESG marginal cost increases, the optimal ESG incentive factor P^s also increases, while the optimal ESG incentive factor P^* remains unchanged. This implies that as the synergistic governance effect decreases the synergistic ESG marginal cost, government policymakers no longer need to offer high incentives to encourage companies to improve ESG performance.



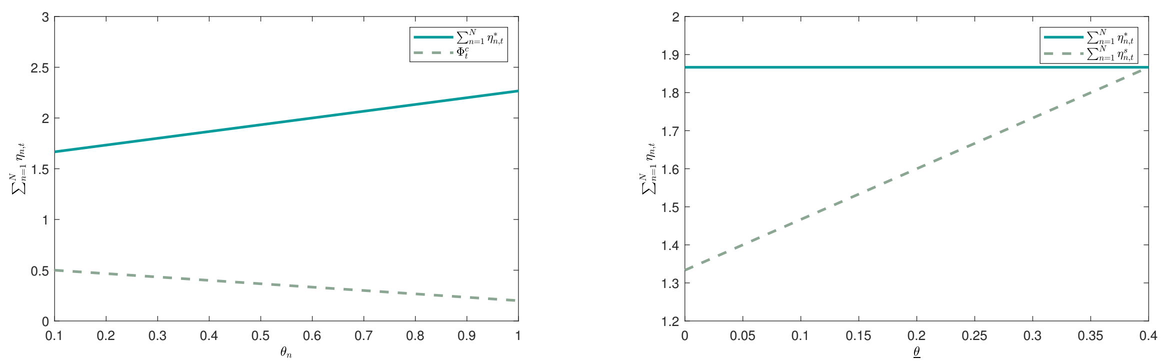
(a) P^* and P^c vs. θ_n (b) P^* and P^s vs. $\underline{\theta}$

Figure 5. Model comparison: (a)Collusive fraud effect: Optimal ESG incentive factor P^* and P^c varies with the margin cost of ESG performance θ_n. (b) Synergistic governance effect: Optimal ESG incentive factor P^* and P^s varies with the margin cost of ESG performance $\underline{\theta}$.

Figure 6(a) depicts the numerical solution of the total ESG performance in a competitive market and under common ownership, resulting in a collusive fraud effect. The figure shows that the total ESG performance $\sum_{n=1}^{N} \eta_{n,t}^*$ in a competitive market is always higher than the total ESG performance Φ_t^c in our chosen parameter settings, where common ownership leads to a collusive fraud effect. In competitive markets, as the marginal cost of a firm's ESG performance increases, the total ESG performance increases due to the higher optimal ESG incentive factor P^*. However, in the case of government intervention, the total ESG performance reduces due to the collusive fraud effect of common ownership. In co-owned firms, as the marginal cost of ESG performance increases, the total ESG performance decreases due to the collusive fraud effect. Moreover, the collusive fraud effect of common ownership cause a higher reduction in the total ESG performance $\sum_{n=1}^{N} \eta_{n,t}^* - \Phi_t^c$ with an increase in the marginal cost of ESG performance.

Figure 6(b) depicts the numerical solution for the total ESG performance in a competitive market and under common ownership, resulting in a synergistic governance effect. The figure reveals that, in our chosen parameter settings, the total ESG performance $\sum_{n=1}^{N} \eta_{n,t}^*$

is consistently higher in a competitive market than the total ESG performance $\sum_{n=1}^{N} \eta_{n,t}^{s}$ in co-owned firms, where common ownership leads to a synergistic governance effect. As the synergistic ESG marginal cost increases in co-owned firms, the total ESG performance rises. However, when the synergistic ESG marginal cost reaches the ESG performance cost in a competitive market, common ownership no longer results in an improvement in total ESG performance. This indicates that, with a collaborative governance effect of common ownership, the optimal decision of the government leads to a decline in the total ESG performance of the market, rendering the over-optimization behavior of government decision-makers unnecessary.



(a) $\sum_{n=1}^{N} \eta_{n,t}^{*}$ and Φ^{c} vs. θ_n (b) $\sum_{n=1}^{N} \eta_{n,t}^{*}$ and $\sum_{n=1}^{N} \eta_{n,t}^{s}$ vs. $\underline{\theta}$

Figure 6. Model comparison: (a)Collusive fraud effect: Total ESG performance $\sum_{n=1}^{N} \eta_{n,t}^{*}$ and Φ^{c} varies with the margin cost of ESG performance θ_n. (b) Synergistic governance effect: Total ESG performance $\sum_{n=1}^{N} \eta_{n,t}^{*}$ and $\sum_{n=1}^{N} \eta_{n,t}^{s}$ varies with the margin cost of ESG performance $\underline{\theta}$.

One of the most contentious issues across all types of ESG literature is whether corporate responsibility management choices have an impact on company performance and value, and whether ESG choices are driven by performance or valuation. Recent financial research, such as (Pástor et al., 2021; Riedl and Smeets, 2017; Barber et al., 2021; Krueger et al., 2020), indicates that some investment groups are willing to sacrifice financial returns for improvements in ESG performance. Some literature suggests that common institutional investors can facilitate collusion among firms to increase market share and bargaining power, leading to higher returns. In such cases, due to decreased market competition intensity, firms no longer need to rely on improving their ESG performance to enhance their market competitiveness (Cheng et al., 2022). These empirical studies' findings are consistent with our theoretical model's conclusions.

7. Conclusion

We examine the design of incentive securities related to ESG performance within a continuous-time principal-agent model, where a company's project operations create negative externalities. In contrast to prior research, we introduce dynamic processes of externality that are affected by decision-makers' ESG performance in a traditional principal-agent framework. In a competitive market, government policymaker influence companies' decision-making through competitive ESG incentive policies to encourage them to improve their ESG performance spontaneously. We then construct a multi-player game model consisting of government policymakers and N firms, including principals and agents.

In our foundational model, the behavior of participants affects project cash flows, with a particular focus on the influence of agents' private efforts, as extensively researched in the security design literature. However, contract theory seldom considers externalities, let alone a company's ESG performance as a decision variable. We propose that the agent has the ability to shape the firm's ESG performance, which indirectly impacts the average growth rate of project cash flows and the accumulation of externalities. Government policymakers provide competitive ESG incentives, and policy and competition determine the impact of ESG performance on a company's cash flow.

To tackle this issue, we apply the stochastic maximum principle method to determine optimal solutions for workload recommendation, compensation payment, and incentive problems. We use a second-order ordinary differential equation to depict the contractual incentive relationship within the enterprise. For external factors, we solve the industry's competition problem and the government's policy-making problem using game theory. We provide explicit expressions for the equilibrium ESG performance and optimal ESG incentive factor of competitive firms.

The optimal contract derived from complex contract theory is often abstract and challenging to implement directly in practical settings. Therefore, we extend our analysis beyond the theoretical model to include considerations of equity, long-term bonds, and credit line securities portfolios to fully realize the optimal contract. The credit lines function as a "ledger" to monitor the continuation value of the agent. Additionally, we provide a concrete application example based on the latest government policies and potential scenarios to establish a scientific foundation for policy formulation and implementation.

We consider two effects of institutional investors' common ownership: the collusive fraud effect and the synergistic governance effect. When government ESG incentives are fixed, the collusionary fraud effect may either increase or decrease total ESG performance, depending on the cost of ESG performance and the relative size of the incentive. The synergistic governance effect, on the other hand, leads to an increase in total ESG performance. However, when the government adopts the theoretical optimal ESG incentive policy, both the collusive fraud effect and the synergistic governance effect lead to a decline in total ESG performance. Thus, the government's formulation of the optimal ESG incentive policy becomes "unnecessary" at this point. By comparing static analysis, we provide a more rigorous verification of our conclusion.

For simplicity, we utilize a multi-player game model in which managers make only three behavioral choices: ESG performance, effort, and consumption. However, in reality, participants hedge the risk of project cash flows and fluctuations of externalities resulting from project operations by investing in liquid financial markets. Additional investment tools can enhance the welfare of participants and make project risk management more interesting and challenging. Furthermore, we do not consider cases where ESG performance is unobservable, which we will address in future research.

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Appendix A. Proof of securities implementation

Proof. The stochastic differential equation governing the company's credit line drawdown balance M_t is as follows:

$$\mathrm{d}M_t = rM_t\mathrm{d}t + k\mathrm{d}t + Div_t\mathrm{d}t - \mathrm{d}X_t, \tag{A.1}$$

where Div_t denotes the dividend paid.

By combining (35) and (36) with (34), we can obtain the value of the agent's continuation value that corresponds to the balance of credit line withdrawals M_t:

$$\begin{aligned}
\mathrm{d}W_t &= -\frac{\lambda}{1-\tau}\mathrm{d}M_t \\
&= -\frac{\lambda}{1-\tau}\left(rM_t\mathrm{d}t + k\mathrm{d}t + \mathrm{d}Div_t - (1-\tau)\mathrm{d}X_t\right) \\
&= \left(\gamma W_t - \frac{\lambda}{1-\tau}Div_t\right)\mathrm{d}t + \lambda\left(\mathrm{d}X_t - \bar{a}\mathrm{d}t - \Pi_n\mathrm{d}t\right).
\end{aligned} \tag{A.2}$$

Denote $\lambda Div_t = c_t - \lambda\bar{a}$. At this point, (A.2) equals the value of the agent's commitment that corresponds to the optimal contract. The condition $\lambda Div_t = c_t - \lambda\bar{a}$ indicates that the dividend received by the agent on the portion of internal equity held corresponds to their compensation utility net of the cost of effort. □

Appendix B. Proofs of lemmas and propositions

Proof of Prop. 3.3. The first-order condition for problem (8) indicates that the optimal effort behavior $a^*_{n,t}$ of the professional manager can be expressed as:

$$a^*_{n,t} = \begin{cases} \bar{a}, & \alpha_{n,t} \geqslant \lambda, \\ 0, & \alpha_{n,t} < \lambda. \end{cases} \tag{B.1}$$

Similarly, the first-order condition implies that the optimal ESG responsibility performance $\eta^*_{n,t}$ of the manager can be expressed as:

$$P - Q\sum_{m\neq n}\eta_{m,t} - 2Q\eta_{n,t} - \theta_n = 0. \tag{B.2}$$

Therefore, we can arrive at the aforementioned conclusion. □

Proof of Prop. 3.4. We simultaneously solve the following equations for $\eta^*_{1,t}$ and $\eta^*_{2,t}$:

$$\begin{cases} P - Q\eta_{2,t} - 2Q\eta_{1,t} - \theta_1 = 0, \\ P - Q\eta_{1,t} - 2Q\eta_{2,t} - \theta_2 = 0. \end{cases} \tag{B.3}$$

Further we can come to the above conclusion. □

Proof of Prop. 3.5. Substituting (9) into following conditions:

$$\left(P - Q\sum_{m=1}^{N} \eta^*_{m,t}\right) \eta^*_{n,t} \geqslant 0,\ \ n = 1, 2, \tag{B.4}$$

and

$$\eta^*_{n,t} \geqslant 0,\ \ n = 1, 2. \tag{B.5}$$

Further we can come to the above conclusion. □

Proof of Prop. 3.6. It is worth noting that the agent possesses a perpetual American option. Thus, we can obtain the result by comparing the waiting value W_n with the exercising value R_n in the usual manner. □

Proof of Prop. 4.2. The first-order condition for problem (8) indicates that the optimal effort behavior $a^c_{n,t}$ of the agent can be expressed as:

$$a^c_{n,t} = \begin{cases} \bar{a}, & \alpha_t \geqslant \lambda, \\ 0, & \alpha_t < \lambda. \end{cases} \tag{B.6}$$

Similarly, the first-order condition indicates the agent's optimal ESG performance $\eta^c_{n,t}$ can be expressed as

$$P - 2Q\Phi_t - \frac{1}{N}\sum_{n=1}^{N} \theta_n = 0. \tag{B.7}$$

Further we can come to the above conclusion. □